

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT3

Branch: B. Tech (All)

Semester : 2

Date : 20/04/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) The series $1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \dots \infty$ is
 (A) convergent (B) divergent (C) finitely oscillating
 (D) infinitely oscillating
- b) The interval of convergence of the logarithmic series
 $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \infty$ is
 (A) $-1 < x \leq 1$ (B) $-1 < x < 2$ (C) $-\infty < x < \infty$ (D) $-1 \leq x \leq 1$
- c) The value of $\int_{-1}^1 \sin^{11} x \, dx$
 (A) 10! (B) $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{5} \cdot \frac{\pi}{2}$ (C) 0 (D) none of these
- d) If $f_n = \int_0^{\pi/4} \tan^n x \, dx$, then $(f_n + f_{n-2})$ is equal to?
 (A) $\frac{1}{n}$ (B) $\frac{1}{n-1}$ (C) $\frac{n}{n-1}$ (D) $\frac{n-1}{n}$
- e) $\int_1^{\infty} \frac{1}{x^{\sqrt{2}}} \, dx$ is convergent.
 (A) True (B) False
- f) $\sqrt{\frac{1}{2}} \sqrt{\frac{3}{2}} \sqrt{\frac{5}{2}} = \underline{\hspace{2cm}}$
 (A) $\frac{3}{8}(\pi)^{\frac{3}{2}}$ (B) $\frac{3}{8}(\pi)^{\frac{5}{2}}$ (C) $\frac{3}{8}(\pi)^{\frac{1}{2}}$ (D) $\frac{1}{8}(\pi)^{\frac{3}{2}}$
- g) $B(1, 1) = \underline{\hspace{2cm}}$
 (A) 1 (B) 0 (C) 1/2 (D) none of these
- h) The tangents at the origin are obtained by equating to zero
 (A) the lowest degree terms (B) the highest degree terms



- (C) constant term (D) none of these
- i) $\int_0^a \int_0^{\sqrt{a^2-y^2}} dx dy$ is equal to
 (A) πa^2 (B) $\frac{\pi a^2}{2}$ (C) $\frac{\pi a^2}{4}$ (D) none of these
- j) $\int_0^{\frac{\pi}{2}} \int_0^{\infty} e^{-r^2} \cdot r dr d\theta$ is equal to
 (A) $\frac{\pi}{2}$ (B) π (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$
- k) The transformations $x + y = u$, $x - y = v$ transform the area element $dy dx$ into $|J| du dv$, where $|J|$ is equal to
 (A) $\frac{1}{2}$ (B) 1 (C) u (D) none of these
- l) The degree and order of the differential equation of all parabolas whose axis is x-axis are
 (A) 2, 1 (B) 1, 2 (C) 3, 2 (D) none of these
- m) The solution of the equation $x \frac{dy}{dx} = y + x \tan\left(\frac{y}{x}\right)$ is
 (A) $\sin\left(\frac{x}{y}\right) = cx$ (B) $\sin\left(\frac{y}{x}\right) = cx$ (C) $\sin\left(\frac{x}{y}\right) = cy$ (D) $\sin\left(\frac{y}{x}\right) = cy$
- n) The differential equation of all non-vertical lines in a plane is
 (A) $\frac{d^2 y}{dx^2} = 0$ (B) $\frac{dy}{dx} = 0$ (C) $\frac{dx}{dy} = 0$ (D) $\frac{d^2 x}{dy^2} = 0$

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a) Test the convergence of the series $\frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{4}-1} + \dots$ (5)
- b) Using reduction formula evaluate: $\int_0^{\pi} x \sin^7 x \cos^4 x dx$ (5)
- c) Prove that $\int_0^{\infty} \frac{x^4}{4^x} dx = \frac{24}{(\log 4)^5}$. (4)

Q-3 Attempt all questions (14)

- a) Prove that $\int_0^1 x^5 (1-x^3)^{10} dx = \frac{1}{3} B(2, 11)$. (5)
- b) Using reduction formula prove that $\int_0^{\pi} x \cos^6 x dx = \frac{5\pi^2}{32}$. (5)



c) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2}$. (4)

Q-4 Attempt all questions (14)

a) Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy \, dy \, dx$ and hence (5)

evaluate it.

b) Examine the series $\sum_{n=1}^{\infty} \frac{x^n}{n^p}$ for convergence using root test. (5)

c) Solve: $\frac{(x-2y) \, dy}{(3x+y) \, dx} = 3x^2 - 5xy - 2y^2$ (4)

Q-5 Attempt all questions (14)

a) Solve: $\left(xy^2 + e^{-\frac{1}{x^3}} \right) dx - x^2 y \, dy = 0$ (5)

b) By changing the transformations $x + y = u$, $y = uv$, show that (5)

$$\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} \, dy \, dx = \frac{e-1}{2}.$$

c) Using reduction formula evaluate: $\int_0^1 \frac{x^6}{(1+x^2)} \, dx$ (4)

Q-6 Attempt all questions (14)

a) Prove that $\int_0^{\frac{\pi}{2}} \frac{dx}{\tan^p x} = \frac{\pi}{2} \sec\left(\frac{p\pi}{2}\right)$. (5)

b) Solve: $(x^2 + y^2 + 1)dx - 2xy \, dy = 0$ (5)

c) Evaluate: $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) \, dz \, dy \, dx$ (4)

Q-7 Attempt all questions (14)

a) Trace the curve $xy^2 = 4a^2(2a-x)$. (5)

b) Show that $\int_1^{\infty} \frac{\ln x}{x^2} \, dx$ converges and compute its value. (5)

c) Find the area enclosed by the cardioid $r = a(1 - \cos \theta)$. (4)

Q-8 Attempt all questions (14)

a) Find the volume of the solid generated by the revolution of the loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$. (5)

b) Find the asymptotes of the curve $y^3 - x^2(6-x) = 0$. (5)

c) Evaluate: $\int_0^{\infty} \frac{dv}{(1+v^2)(1+\tan^{-1} v)}$ (4)

